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THE DESIGN OF A CONCRETE-STEEL ARCH BRIDGE

by

HARRY JAMES COWIE

A Thesis submitted for the Degree of
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THE DESIGN OF A CONCRETE-STEEL ARCH BRIDGE.

The bridge herein designed is one to replace what is known as the "Rustic Bridge," located on the west side of Eagle Heights where the Pleasure Drive around Lake Mendota crosses a deep ravine. The conditions are very favorable for the construction of an arch bridge, since there is plenty of head room and the sides of the ravine are composed of sandstone which will take the thrust of the arch. Such a type of bridge will also be very suitable to the topography as it is a heavy and substantial form of construction well in keeping with the high wooded hill back of it, the rough and rocky side of the ravine and the steep rocky shore of the lake to the north.

The topography for a short distance about the location is shown on Plate I and a profile through the center of the bridge in Figure 1. From these conditions the rise of seven and a half feet, and the span of seventy feet, were determined. The small rise in comparison with the length of span was considered advisable on account of the rock side walls which could resist all the thrust and a much smaller fill would be required above the arch.

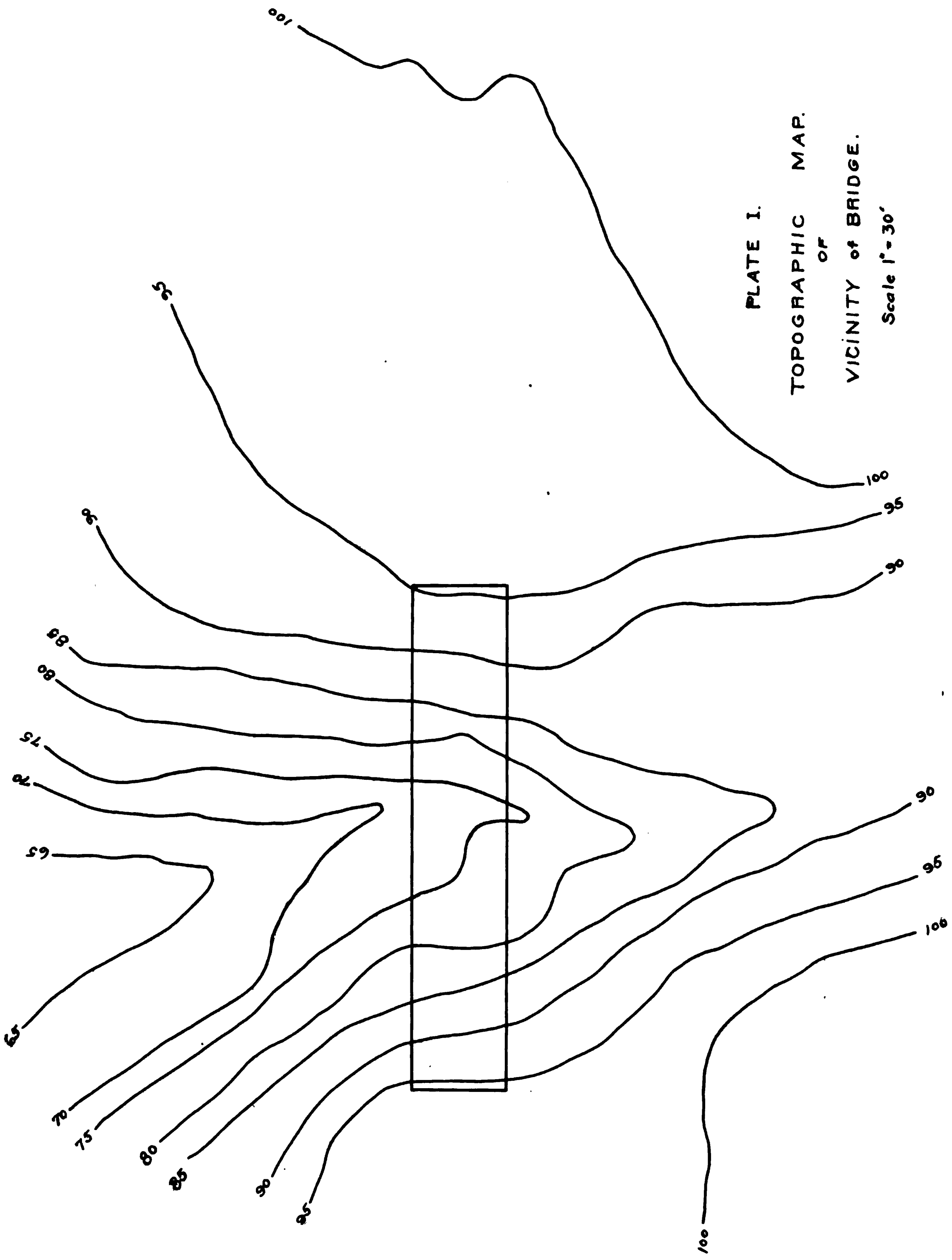
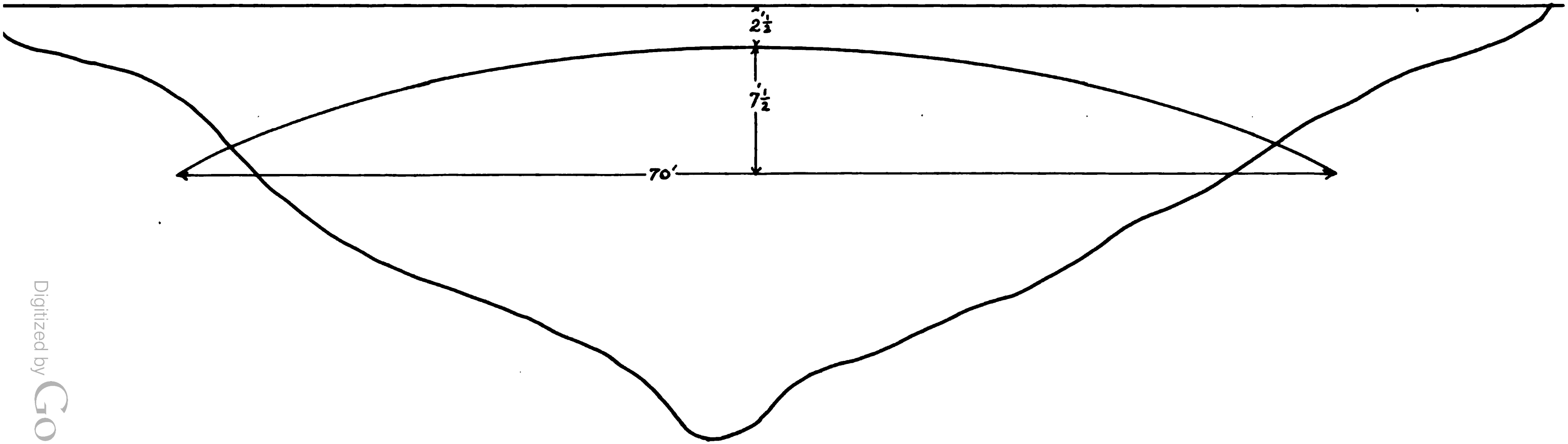


PLATE I.
TOPOGRAPHIC MAP
OF
VICINITY of BRIDGE.
Scale 1" = 30'

Fig. 1

PROFILE THROUGH CENTER OF BRIDGE.

Scale 1 In. = 10 Ft.



The method followed in the design is based on the principle that the most economical form of arch ring is that of a linear arch for the given loading. The pressure cannot be determined but the two limiting conditions are known. Curves of equilibrium can be located for these two cases considering first that the earth pressures are vertical and, second, that they are normal to the extrados of the arch. Every condition of earth loading must lie between these two extremes. If for the first condition of loading the pressures be considered as proportional to the depth of filling above, the curve of equilibrium will be the catenary or its vertical projection the vertical catenary. If for the same condition the pressures be considered as normal to the transformal catenary, the curve of equilibrium will be an approximation to the hydrostatic arch.

The arch was then assumed as hinged at the crown, making the two curves of equilibrium tangent at that point. They diverge as they approach the abutments, the transformed catenary lying above. The curve of equilibrium for vertical pressures is first determined and the extrados drawn nearly parallel to it. The pressures are then assumed normal to the extrados and the curve of equilibrium drawn for this condition of loading. The intrados of the arch is now drawn parallel to this curve.

The curve of equilibrium for vertical pressures can

be determined analytically by reducing the vertical ordinates of the catenary by a fixed ratio. The equation of the catenary referred to its directrix as axis is $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$ when a is the least ordinate or parameter of the catenary and e is the base of the Naparian system of logarithms or 2.71828. The equation of the transformal catenary is then $y = \frac{y_0}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$ where y_0 is the least ordinate of the transformal catenary. Expressed in hyperbolic functions the equation is $y = y_0 \cosh \frac{x}{a}$. The radius of curvature of any curve is given by $R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$.

Differentiating the equation of the transformal catenary, $\frac{dy}{dx} = -\frac{y_0}{a} \sinh \frac{x}{a}$ and $\frac{d^2y}{dx^2} = -\frac{y_0}{a^2} \cosh \frac{x}{a}$. Substitute these values and neglect the minus sign, then

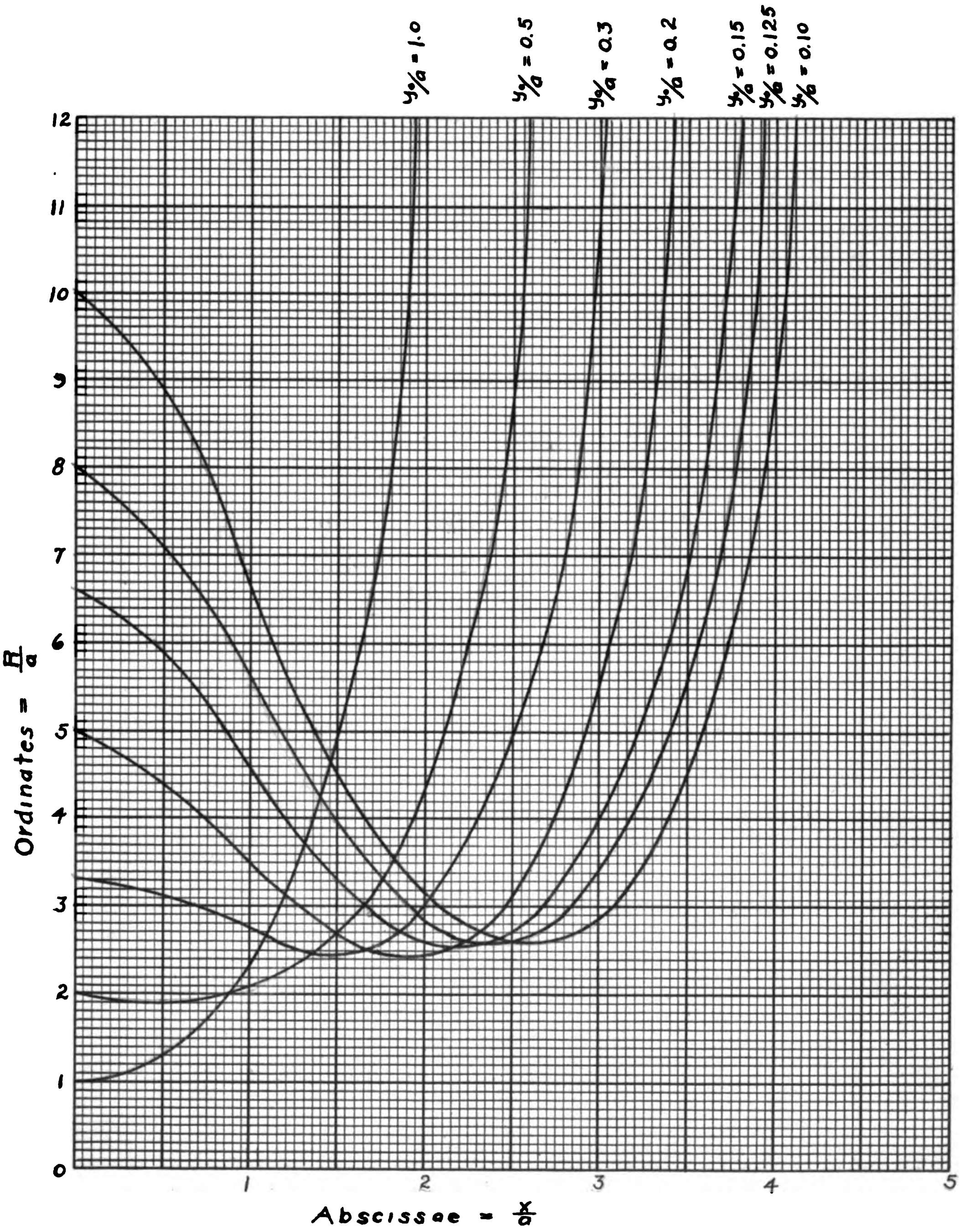
$$R = \frac{\left[1 + \frac{y_0^2}{a^2} \sinh^2 \frac{x}{a}\right]^{\frac{3}{2}}}{\frac{y_0}{a^2} \cosh \frac{x}{A}}$$

$$\text{or} \quad \frac{R}{a} = \frac{\left[1 + \frac{y_0^2}{a^2} \sinh^2 \frac{x}{a}\right]^{\frac{3}{2}}}{\frac{y_0}{a} \cosh \frac{x}{a}}$$

Assuming definite values for $\frac{y_0}{a}$ and solving for $\frac{R}{a}$ the former may be plotted as abscissas and the latter as ordinates the curves shown in Figure 2 may be obtained. The radius of

Fig. 2.

Radii of Curvature of Transformed Catenary.



curvature can be obtained from this diagram if the value of $\frac{y_0}{a}$ is known. The value of a depends upon the span, rise and depth of filling at the crown. Its value may be determined from the equation

$$a = \frac{x_1}{\text{hyp. log}\left[\frac{y_1}{y_0} + \sqrt{\left(\frac{y_1}{y_0}\right)^2 - 1}\right]}$$

when x_1 is the abscissa of any point through which it is desired the transformal catenary to pass and y_1 is the ordinate of the same point. The point x_1, y_1 is usually the springing point of the center line of the arch. Then x_1 equals the half span and y_1 equals the depth of fill over the springing point. Then the value of $\frac{y_0}{a}$ is given directly by the equation

$$\frac{y_0}{a} = \frac{y_0}{x_1} \text{ hyp. log}\left[\frac{y_1}{y_0} + \sqrt{\left(\frac{y_1}{y_0}\right)^2 - 1}\right]$$

Applying these formulas to the arch with a span of 70 feet, rise of 7-1/2 feet and depth of loading at the crown of 2-1/2 feet, x_1 equals 35 feet, y_1 equals 10 feet and y_0 equals 2-1/2 feet. Then $\frac{y_0}{a} = \frac{2.5}{35} \text{ hyp. log}\left[\frac{10}{2.5} + \sqrt{\left(\frac{10}{2.5}\right)^2 - 1}\right]$ or $\frac{y_0}{a} = .147$, and $\frac{2.5}{a} = .147$ or $a = 17$. From Figure 1 the values of R for each ten foot length of arch rib may now be obtained.

x	$\frac{x}{a}$	$\frac{R}{a}$	R
0	0	6.8	115.5
10	.59	5.9	100.2
20	1.18	4.2	71.4
30	1.76	4.0	68.o
35	2.06	2.6	44.2

Using a three-centered arch and averaging the above values of R, the radius of curvature for 20 feet each side of the center will be 95.7 feet; from 20 to 30, 69.7 feet, and the last 5 feet, 56.2 feet.

Making the intrados of the arch three-centered also, the radii of curvature can be determined from the fact that at any point it is to the radius at the center inversely as the vertical height of the load above those points. Assuming that at the center the intrados is concentric with the extrados, its radius of curvature at that point will be .75 feet less, or 114 feet. The radius of curvature at each ten foot length of the arch may now be determined. Let x equal the distance from the center, y the vertical height of the load above and R the radius of curvature.

x	y	R
0	2.40	113.5
10	2.75	99.4
20	3.80	71.9
30	6.20	44.1
35	8.00	34.2

Averaging these values the radius of curvature 20 feet each side of the center is 95.1 feet; from 20 to 30 feet, 58.0 feet, and the last 5 feet, 39.1 feet.

The curves of equilibrium for both conditions of loading can also be determined graphically. On the left in Plate II is derived the curve for the vertical pressures. The three known points are the springing points, and crown of the arch rib. The curve of equilibrium is the catenary, but since a catenary in general can not be made to pass through three points it must be transformed. The curve bc is the catenary which was determined for this arch. The distance ab was layed off equal to a, which was previously found to equal 17. It also represents the thrust AB, provided the division of the load line be made equal to those of the arc of the catenary. The load line was divided into a convenient number of equal parts commencing with a half division at B. The elements of the curve bc were now drawn parallel to the successive rays of the force polygon and the

lengths made equal to those on BC. The transformed catenary was divided from this by reducing each vertical ordinate in the ratio of 815 to 120. This was done graphically by assuming two points n and m at a distance from the directix of 815 and 120 respectively. Lines were then drawn from n through points on the catenary to an intersection with the directrix and from these points back to the point m. This line will divide the vertical ordinates in the given ratio and locate points on the transformed catenary a c . The horizontal thrust is the same for the transformed catenary as for the catenary. The thickness of the arch ring at the crown was assumed at 18 inches and the extrados drawn parallel to the transformed catenary at a distance of 9 inches from it. The thickness should have been increased towards the abutments, proportional to the thrust, but since the arch was very flat the difference is comparatively small.

On the right of Plate II the curve of equilibrium for normal pressures is determined. By spacing the points of application at the same horizontal intervals as in the preceding case, the normal pressures will be equal and the load line may be drawn with elements of equal length. Assuming the same horizontal thrust as in the transformed catenary, the point D is obtained by drawing a line CD parallel to MA. The elements of the curve of equilibrium, fg, are then drawn parallel to the rays of the force polygon and limited in length by the points of intersection with the normal forces.

The intrados is drawn parallel to and at a distance of 9 inches from fg. The resulting form of the arch ring is shown on the right of the plate.

The arch was now tested by the theory of the elastic arch which was shown by the tests of the Austrian Society of Engineers and Architects to be the only solid foundation for theoretical investigation since the deformations were practically proportional to the stress in all cases where the elastic limit was not exceeded. Calculations for thrusts, bending moments and shears can be made graphically or analytically, but the latter method is too complicated and laborious to be considered.

A section of the arch one foot wide was taken upon which to make the test. The quantity of steel assumed was one percent of the cross-section at the crown, placed three inches from the extrados and intrados in the form of bars one inch thick. Tests were made for three conditions of loading; first, for dead load only, second, with a uniform live load of 100 pounds per foot over the bridge, and, third, with a fifteen ton steam roller with one set of rolls placed at the center.

Taking up the dead load first: on Plate III the gravity axis, which is the center of the arch rib in this case since the section is symmetrical, was divided into eighteen equal parts, each of which was called ΔS. Vertical lines

were then drawn through the extremities and the centers of each ΔS and the load P considered as acting along the latter lines. The reduced dead load contour AB was drawn assuming the weight of the concrete to be 140 pounds per cubic foot and the weight of the earth 100 pounds. The moment of inertia for each section taken radially through the center of each ΔS was computed as shown in Table 1. The moment of inertia of the steel was multiplied by 19 to reduce it to an equivalent for concrete, and the ratio of the moment of inertia at the crown and that at any other point or $\frac{1}{n}$ determined.

The arch must be treated as one with fixed ends and no hinges, with immovable piers and as symmetrical about a vertical line. Such an arch rib is shown in Figure 3, with the span OB taken as the axis of x. The co-ordinates of any point n on the arch rib are x and y, and z is the vertical intercept between n and the special equilibrium polygon. The special equilibrium polygon must satisfy the three conditions*

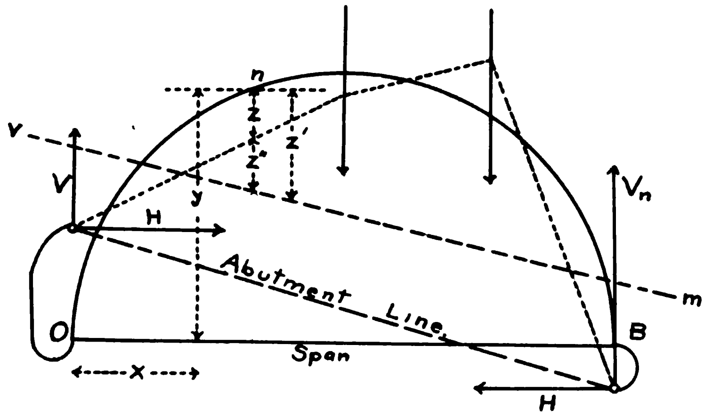


Figure 3.

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*Church, Mechanics of Engineering.

$$\int_0^B \frac{M ds}{E\ I} = 0; \qquad \int_0^B \frac{M y ds}{E\ I} = 0; \qquad \int_0^B \frac{M x ds}{E\ I} = 0$$

As E is a constant and M = Hz the conditions may be stated

$$\int_0^B \frac{z}{n} ds = 0; \qquad \int_0^B \frac{xz}{n} ds = 0; \qquad \int_0^B \frac{yz}{n} ds = 0.$$

Now draw the line vm to satisfy the conditions

$$\int_0^B \frac{z'}{n} ds = 0; \quad \text{and} \quad \int_0^B \frac{xz'}{n} ds = 0,$$

when z' is the vertical distance of any point n from the line vm. Let z'' denote the vertical intercept between the special equilibrium polygon and the line vm. Then $z = z' + z''$ and the three conditions become

$$\int_0^B \frac{z' - z''}{n} ds = 0; \quad \text{or} \quad \int_0^B \frac{z''}{n} ds = 0$$

$$\int_0^B \frac{x(z' - z'')}{n} ds = 0; \quad \text{or} \quad \int_0^B \frac{xz''}{n} ds = 0 \quad \text{and}$$

$$\int_0^B \frac{y(z' - z'')}{n} ds = 0; \quad \text{or by transposition}$$

$$\int_0^B \frac{yz'}{n} ds = \int_0^B \frac{yz''}{n} ds .$$

As the arch rib has been divided into an equal number of equal parts and a vertical drawn through the middle of each, vm may be located to satisfy the conditions.

$$\sum_0^B \left(\frac{z'}{n}\right) = 0 \quad \text{and} \quad \sum_0^B \frac{xz}{n} = 0.$$

The special equilibrium polygon can then be located with vm as a reference line by making it satisfy the conditions

$$\sum_0^B\left(\frac{z''}{n}\right) = 0\ ; \qquad \sum_0^B\left(\frac{xz''}{n}\right) = 0\ ; \qquad \text{and} \quad \sum_0^B\left(\frac{yz''}{n}\right) = \sum_0^B\left(\frac{yz'}{n}\right).$$

From symmetry vm must be horizontal so a trial line is drawn and $\frac{z'}{n}$ summed up as shown in Table 2. As the $\sum_1^9 +\left(\frac{z'}{n}\right)$ exceeds $\sum_0^B\left(-\frac{z'}{n}\right)$ by z_o the line must be raised by an amount $\frac{z_o}{\sum_1^9\left(\frac{1}{n}\right)}$ or $\frac{2.66}{7.118} = .338$. The summation of the same qualities with respect to the new vm is also given in Table 2.

The load line is layed off with the vertical distance between the gravity axis and the dead load contour, reduced one half. A trial pole O_1''' is selected and the trial equilibrium polygon F''' G''' drawn. In F''' G''' a horizontal line, v''' m''' is located in the same manner as vm, the summation being shown in Table 3. Z_o equals + 1.5, in this case and v''' m''' must be lowered 0.21. Project the intersection E''' of v''' m''' with F''' G''' vertically upward to an intersection, E, with vm which is one point through which the special equilibrium polygon must pass.

The ratio must now be determined by which H''' must be altered in order to fulfill the condition

$\sum_1^{18}\left(\frac{yz'''}{n}\right) = \sum_1^{18}\left(\frac{yz'}{n}\right)$ Lay off $\frac{z'}{n}$ as a load line pay-

TABLE 1.

For sections cut by the forces P.

Section	Depth	I_c	$I_s \times 19$	Total I	$\frac{1}{n} = \frac{I_o}{I}$
1	23.5	12978	3212	16190	.454
2	21	9261	2360	11621	.630
3	20	8000	2058	10058	.730
4	19.5	7620	1963	9583	.765
5	19	7227	1854	9081	.807
6	18.8	6851	1734	8585	.855
7	18.5	6331	1638	7969	.920
8	18.25	6078	1580	7658	.957
9	18	5832	1512	7344	1.000

TABLE 2.

Section	$\frac{1}{n}$	Trial vm			Final vm		
		Z'	$+\frac{Z'}{n}$	$-\frac{Z'}{n}$	Z'	$+\frac{Z'}{n}$	$-\frac{Z'}{n}$
1	.454	+ 44.2	2.02		+ 4.92	2.23	
2	.630	+ 2.80	1.76		+ 2.27	1.43	
3	.730	+ 1.32	.96		+ 1.82	1.33	
4	.765	+ .20	.15		+ .70	.50	
5	.807	− .62		.50	− .15		.12
6	.855	− 1.30		1.11	− .80		.68
7	.920	− 1.78		1.64	− 1.30		1.20
8	.957	− 2.10		2.06	− 1.63		1.56
9	1.000	− 2.24		2.24	− 1.76		1.76

TABLE 3.

DEAD LOAD.

Section	$\frac{1}{n}$	Trial $v'''m'''$			Final $v'''m'''$		
		Z'''	$+\frac{Z'''}{n}$	$-\frac{Z'''}{n}$	Z'''	$+\frac{Z'''}{n}$	$-\frac{Z'''}{n}$
1	.454	+ 4.55	2.06		+ 4.31	2.00	
2	.630	+ 3.06	1.93		+ 2.80	1.76	
3	.730	+ 1.80	1.31		+ 1.60	1.17	
4	.765	+ .80	.61		+ .59	.45	
5	.807	0	0		− .21		.17
6	.855	− .60		.51	− .83		.71
7	.920	− 1.10		1.01	− 1.30		1.20
8	.957	− 1.40		1.34	− 1.60		1.53
9	1.000	− 1.55		1.55	− 1.75		1.75
			5.91	4.41		5.38	5.36

TABLE 4.

UNIFORM LIVE LOAD.

Section	$\frac{1}{n}$	Trial $v''m''$			Final $v''m''$		
		Z''	$+\frac{Z''}{n}$	$-\frac{Z''}{n}$	Z''	$+\frac{Z''}{n}$	$-\frac{Z''}{n}$
1	+ .454	+ 5.10	2.32		+ 4.90	2.22	
2	.630	+ 3.45	2.18		+ 3.20	2.02	
3	.730	+ 2.02	1.48		+ 1.80	1.31	
4	.765	+ .87	.67		+ .62	.47	
5	.807	− .11		.09	− .30		.24
6	.855	− .80		.68	− 1.00		.85
7	.920	− 1.31		1.21	− 1.52		1.40
8	.957	− 1.67		1.60	− 1.88		1.80
9	1.000	− 1.82		1.82	− 2.04		2.04
			6.65	5.40		6.02	6.33

ing due attention to the signs and choose a convenient pole O^{IV} with a pole distance H_o, as shown in Figure (a). $\frac{z'}{n}$ in each case is multiplied by two. The y's are also laid off to twice the scale as shown in Figure (a') and the equilibrium polygon drawn to determine the closing side K. Then

$\sum_1^9 \left(\frac{yz'}{n}\right) - H_o K$. The same operation is gone through with $\left(\frac{z'''}{n}\right)$ and the same pole distance H_o in Figure (b) and the closing side K determined in Figure (b'). Then

$\sum_1^9 \left(\frac{yz''}{n}\right) = H_o K_D$ Since H_oK does not equal H_oK_D the true pole distance will be $H = \frac{K}{K_D} \cdot H'''$ or $H = \frac{5.8}{5.9}$ 441 = 43. With the new pole O''' the special equilibrium polygon is drawn within the arch ring, beginning at the point E, through which it must pass.

In the same way as for the dead load the live load line is laid off and a convenient pole O'' chosen. The trial equilibrium polygon $F''G''$ is drawn and the line $v''m''$ located as before, the results being shown in Table 4. $Z_o = + 1.25$ and $\frac{Z_o}{\sum\left(\frac{1}{n}\right)} = \frac{1.25}{7.118}$ or the line must be lowered .175.

The point E'' is projected vertically upward to an intersection with vm locating the point E through which the special equilibrium polygon must pass.

The true pole distance must now be determined. $\frac{z''}{n}$ is layed off as before, as shown in Figure (c), and the equilibriu polygon drawn in Figure (c') determining the closing side

K_u. The true pole distance H is then equal to $\frac{K}{K_u}$ H'' or $\frac{5.8}{6.2}$ 45 = 42.1. With the new pole 0 the force polygon is completed and the special equilibrium polygon drawn within the arch ring beginning at the point E.

For a concentrated load it was assumed that a fifteen ton steam roller would be the maximum which would probably come on the bridge and that the maximum bending moments would be caused by such a load. The rear rolls were placed at the center of the bridge and considered as acting along the two load lines at each side but as the front roll came over a load line it was considered as acting along that one. Draw the load line and choose a convenient pole $0'$ and draw the trial equilibrium polygon $F^{IV}G^{IV}$ and a trial $v^{IV}m^{IV}$. The condition $\sum_1^{18}\left(\frac{z^{IV}}{n}\right) = 0$ must first be satisfied and the computations for it are found in Table 5. Z_o is found to be equal to 3.27 and $\frac{z_o}{\sum_1^{18}\left(\frac{1}{n}\right)} = \frac{3.27}{14.236}$ or the line must be lowered .23 parallel to itself. Draw a line $V^{IV}G^{IV}$. The condition $\sum_1^{18}\left(x\,\frac{z^{IV}}{n}\right) = 0$ must now be satisfied, that is the sum of the moments of the positive $\left(\frac{z^{IV}}{n}\right)$ must equal that of the negative $\left(\frac{z^{IV}}{n}\right)$ or the moment of the resultant of the positive $\left(\frac{z^{IV}}{n}\right)$ must equal that of the resultant of the negative. The ordinates z^{IV} are equal to the difference of the ordinates w and those between the abutment line and $v^{IV}m^{IV}$. Then the condition may be written $\sum_1^{18}\left(\frac{y}{n}\right) = \sum_1^{18}\left(\frac{w}{n}\right)$; or $\sum_1^{18}\left(x\,\frac{y}{n}\right) = \sum_1^{18}\left(x\,\frac{w}{n}\right)$

TABLE 5.

CONCENTRATED LOAD.

Section	$\frac{1}{n}$	Trial $v''m''$			Final $v''m''$		
		z''	$+\frac{z''}{n}$	$-\frac{z''}{n}$	z''	$+\frac{z''}{n}$	$-\frac{z''}{n}$
1	.454	+ 5.30	2.41		+ 5.00	2.27	
2	.630	+ 3.72	2.34		+ 3.42	2.16	
3	.730	+ 2.32	1.69		+ 2.05	1.50	
4	.765	+ 1.12	.86		+ .86	.66	
5	.807	+ .10	.08		− .12		.10
6	.855	− .75		.64	− .95		.81
7	.920	− 1.45		1.33	− 1.68		1.55
8	.957	− 1.85		1.77	− 2.05		1.96
9	1.000	− 2.11		2.11	− 2.33		2.33
9′	1.000	− 2.08		2.08	− 2.24		2.24
8′	.957	− 1.71		1.64	− 1.90		1.82
7′	.920	− 1.22		1.12	− 1.40		1.29
6′	.855	− .60		.51	− .75		.64
5′	.807	+ .17	.14		0		0
4′	.765	+ 1.10	.84		+ 1.00	.76	
3′	.730	+ 2.21	1.61		+ 2.10	1.53	
2′	.630	+ 3.53	2.22		+ 3.40	2.14	
1′	.454	+ 5.03	2.28		+ 4.90	2.20	
			14.47	11.20		13.22	12.74

The line $v^{II}G^{II}$ divides the w ordinates into two sets which will be called u and p. Let R be the resultant of the $(\frac{y}{n})$, T the resultant of the $(\frac{p}{n})$ and T' the resultant of the $(\frac{u}{n})$. Then T + T' must equal R and their resultant must act in the same vertical as R or these quantities considered as forces must act as a balanced system. Lay off as a load line the quantities $(\frac{y}{n})$ in Table 6, and choose a convenient pole as shown in Figure (f). Draw the equilibrium polygon AC and determine R's vertical by projecting the final rays back to intersection.

In the same way lay off the quantities $(\frac{u}{n})$ and $(\frac{p}{n})$ as load lines and chose poles as in Figure (g) and Figure (h). Draw the equilibrium polygon A'C' and A"C" and determine the lines of action of T' and T. R may be considered as a load supported by a horizontal beam with supports in the verticals of T and T', and T and T' as the reactions. The verticals of T and T' are independent of the position of $v^{II}m^{II}$ for if v^{II} be shifted vertically the (p)'s will change in the same proportion while the (u)'s will only be shifted along their verticals and will not change their value. But the value of T is proportional to $V^{II}F^{II}$. Similarly the (u)'s are proportional to $M^{II}G^{II}$ but its vertical remains the same and T is not affected in any way. R is independent of the location of $V^{II}M^{II}$. The true T = Rt+(s+t) and the true T' = Rs+(s+t).

$T = \frac{9.75}{19.25}\ 42.1 = 21.3$ and $T' = \frac{9.5}{19.25}\ 42.1 = 20.8$. Now the

true $V^{II}F^{II} = (T \div T')\ v_2^{II}F_2^{II}$ and $M^{II}G^{II} = (T' \div T')\ M_2^{II}G_2^{II}$ or $V^{II}F^{II} = \frac{21.3}{21.61}\ 6.08 = 6.00$ and $M^{II}G^{II} = \frac{20.8}{20.52}\ 5.80 = 5.89$. Lay these off and draw in the new $V^{II}M^{II}$. Project the points L' and D' vertically upward to intersections with VM and locate the points L and D through which the special equilibrium polygon must pass.

In the force polygon draw O'N parallel to the abutment line in the trial equilibrium polygon $F^{II}G^{II}$ and through N draw a horizontal line. Then the true pole must lie some where on this line. The ratio by which the pole distance H^{II} must be changed must now be determined. Laying off as load lines the quantities $(\frac{z^{II}}{n})$ in Table 5, as shown in Figures (d) and (e), with the values doubled. Draw the equilibrium polygons corresponding as in Figures (d') and (e'), and determine the closing sides K_L and K_R. Then the

$$\sum_1^9 \left(y\frac{z^{II}}{n}\right) = H_0K_L \text{ and } \sum_{1'}^{9'} \left(y\frac{z^{II}}{n}\right) = K_0K_R. \text{ The}$$

$$\sum_1^{18} \left(y\frac{z^{II}}{n}\right) = H_0(K_L+K_R).$$

If the condition $H_0(K_L+K_R) = 2\,H_0K$ is not satisfied, the true pole distance for the special equilibrium polygon will be $H = \frac{K_L+K_R}{2K} \cdot H^{II}$.

$$\text{or } H = \frac{7.3 + 6.0}{2 \times 5.8} \times 49.5 = 56.7.$$

Measure off this distance on the horizontal line through N and complete the force polygon for the special equilibrium polygon. This may now be drawn in the arch ring, beginning at either of the points L or D with the proper ray, and as a check it should always pass through both points.

TABLE 6.

CONCENTRATED LOAD.

Section	$\frac{1}{n}$	R		T		T′	
		Y	$\frac{y}{n}$	p	$\frac{p}{n}$	u	$\frac{u}{n}$
1	.454	.95	.43	5.87	2.66	.14	.06
2	.630	2.52	1.59	5.55	3.50	.46	.29
3	.730	3.90	2.84	5.20	3.80	.78	.57
4	.765	5.07	3.88	4.90	3.75	1.10	.84
5	.807	6.07	4.90	4.60	3.71	1.40	1.13
6	.855	6.91	5.91	4.25	3.64	1.70	1.47
7	.920	7.62	7.00	3.90	3.59	2.05	1.88
8	.957	8.00	7.65	3.55	3.40	2.40	2.30
9	1.000	8.25	8.25	3.20	3.20	2.70	2.70
9′	1.000	8.20	8.20	2.90	2.90	3.05	3.05
8′	.957	7.81	7.49	2.54	2.43	3.38	3.24
7′	.920	7.35	6.75	2.20	2.02	3.70	3.40
6′	.855	6.70	5.73	1.82	1.56	4.05	3.46
5′	.807	5.90	4.76	1.48	1.20	4.35	3.51
4′	.765	4.93	3.77	1.15	.88	4.68	3.58
3′	.730	3.85	2.81	.80	.58	5.00	3.65
2′	.630	2.50	1.57	.50	.31	5.30	3.34
1′	.454	1.00	.45	.20	.09	5.60	2.57

From the diagrams now obtained the shears, thrusts and bending moments are taken for sections taken radially through the extremities of each ΔS. The shears are the normal components of the force acting at the section and divided by the area gives the shear per square inch, which according to the specifications is limited to 100 pounds per square inch on the concrete. The thrust is the tangential component. The moment at any section is equal to the product of the pole distance H of the force diagram and the vertical intercept between the gravity axis and the special equilibrium polygon. The results are given in the tables for the different conditions of loading and also the stresses in the extreme fibers of the steel and concrete.

The stresses were obtained by the use of formulas published by Mr. Thatcher.*

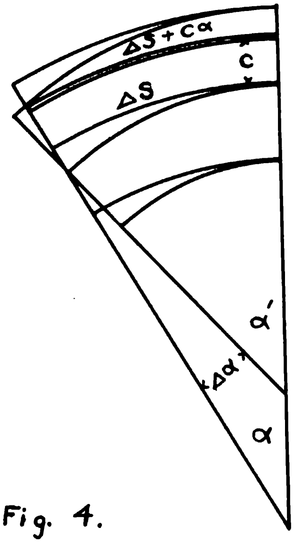

Fig. 4.

In Figure 4 let ΔS be the length of an element of the arch ring, c the distance of a fiber from the neutral axis, α the angle at the center, and α' the changed angle due to the bending moment in the arch. The original length of the fiber is $\Delta S + c\alpha$ before flexure and $\Delta S + c\alpha'$ after flexure. The change of length is

$C(\alpha' - \alpha)$ or $c.\Delta\alpha$. The stress per square inch on the fiber is $f_1 = c_1 E_1 \frac{\Delta\alpha}{\Delta s}$ and $\Delta\alpha = \frac{M \Delta s}{E_1(I_1 + n I_2)}$

Eliminating $\frac{\Delta\alpha}{\Delta s}$ between these two equations

$$f_1 = \frac{M c_1}{(I_1 + n I_2)}$$

and in the same way the stress per square inch in an extreme fiber of the steel at a distance C from the neutral axis is

$$f_2 = \frac{M c_2 n}{(I_1 + n I_2)}$$

These two stresses are due entirely to the bending moment in the arch. There is also a uniform compressive stress over the entire cross-section, due to the thrust, equal to

$$T = P(A_1 + nA_2) \quad \text{or} \quad P = \frac{T}{(A_1 + nA_2)}$$

when P is the compressive stress per square inch and A_1 and A_2 the areas of the concrete and steel respectively. In the same way the uniform compressive stress per square inch on the steel is

$$P_2 = \frac{n\,T}{(A_1 + nA_2)}$$

Combining the stress due to the bending moment and that due to the thrust, the total unit stress S_1 in pounds per square inch exerted on the concrete at the upper or lower edges of the cross-section, is obtained.

$$S_1 = \frac{T}{A_1 + nA_2} \pm \frac{Mc_1}{(I_1 + nI_2)}$$

but $c_1 = \frac{d_1}{2}$ and $n = 20$

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*Church, Mechanics of Engineering, p. 444.

Then $S_1 = \Big[\dfrac{T}{A_1 + 20A_2} \pm \dfrac{d_1 M}{2(I_1 + 20I_2)}$

In the same way the stress S on the steel is obtained.*

$$S_2 = \left[\frac{T}{A_1 + 20A_2} \pm \frac{d_2 M}{2(I_1 + 20I_2)}\right] \times 20$$

(All units in these formulas are in inches and pounds.) The stresses computed by these formulas are given in Tables 8, 9 and 10.

Since the concrete and steel are elastic materials, the thrust will cause a shortening of the arch rib and hence a distortion which will produce additional shears, thrusts and bending moments in the arch. This shortening of the rib may be considered as being caused by a fall of temperature which would have the same effect.

Let P be the average thrust, A the average cross-section, E the modulus of elasticity of concrete, or 1,400,000, and the coefficient of expansion, or .0000066 per degree Fahrenheit.

Then $(t_0 - T) = \dfrac{P}{A E \eta}$

The corresponding changes of temperature for the three conditions of loading are obtained by this formula.

Dead load $(t_o - t) = \dfrac{51060}{240 \times 1{,}400{,}000 \times .0000066} = 23° \text{ F.}$

Uniform live load $(t_o - t) = \dfrac{52700}{240 \times 1{,}400{,}000 \times .0000066} = 23.8° \text{ F.}$

Concentrated load $(t_o - t) = \dfrac{65500}{240 \times 1{,}400{,}000 \times .0000066} = 30° \text{ F.}$

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*Church, Mechanics of Engineering.

TABLE 7.

FOR TEMPERATURE STRESSES.

Section	$\frac{1}{n}$	Y. Ft	$\frac{Y}{n}$ Ft.	$\frac{Y^2}{n}$ Ft.	$\frac{Y}{n}$ Inches	$\frac{Y^2}{n}$ Inches
1	.454	1.05	.47	.49		
2	.630	2.42	1.52	3.69		
3	.730	4.12	3.01	12.35		
4	.765	5.25	4.01	21.10		
5	.807	6.14	4.95	30.20		
6	.855	6.80	5.81	39.50		
7	.920	7.30	6.71	49.00		
8	.957	7.60	7.27	55.20		
9	1.000	7.75	7.75	60.10		
			41.50	271.63	498.0	39114.72

TABLE 8.

DEAD LOAD.

+ Tension — Compression.

Section.	Depth.	Area.	Shears.		Thrust	H	Z	Moments. Inch Lbs.	Stresses.			
									Concrete		Steel.	
			Total.	Per. Sq. In					Upper	Lower	Upper	Lower.
1	26	312	1510	+ 5	55300	49504	5.3	262,000	-138	+ 19	-5180	-340
2	22	264	1960	+ 7	53500	"	3.6	178,000	-300	- 6	-5200	- 920
3	20.5	246	2680	+ 11	52000	"	1.8	89,100	-240	- 72	-4320	-1920
4	20	240	1340	+ 5.6	51200	"	.5	24,800	-185	- 25	- 2540	- 2860
5	19.5	234	840	+ 3.6	50500	"	0	0	-157	-157	-3140	-3140
6	19	228	340	+ 1.5	50000	"	0	0	-153	-153	- 3060	- 3060
7	18.87	226	220	+ 1.0	49700	"	0	0	-220	-220	-4400	-4400
8	18.37	220	450	+ 2.0	49500	"	0	0	-222	-222	-4440	-4440
9	18.2	218	340	+ 1.5	49500	"	0	0	-229	-229	-4580	-4580
10	18	216	0	0	49400	"	0	0	-228	-228	- 4560	-4560

TABLE 9.

UNIFORM LIVE LOAD.

Section	Depth	Area.	Shears.		Thrust.	H	Z	Moments. Inch Lbs.	Stresses.			
									Concrete.		Steel.	
			Total.	Per Sq. In.					Upper	Lower	Upper	Lower.
1	26	312	560	- 2.6	57900	50,510	2.4	121,000	- 73	-145	- 1780	-4020
2	22	264	850	+ 3.2	56600	"	2.1	106000	- 75	-162	- 1980	-4500
3	20.5	246	448	+ 1.8	54000	"	.9	45,500	- 43	-205	-2620	-3860
4	20	240	2180	+ 9.1	52800	"	1.2	60600	- 105	-225	-2460	-4140
5	19.5	234	1010	- 4.3	52000	"	0	0	-222	-222	-4440	- 4440
6	19	228	896	- 3.9	51500	"	1.2	60600	-229	- 99	-4180	-2380
7	18.87	226	190	- .9	51000	"	2.4	121000	-298	- 26	- 5100	-1380
8	18.37	220	450	- 2.0	50800	"	2.0	101,000	-284	- 46	-4900	-1700
9	18.2	218	390	+ 1.9	50,500	"	2.1	106,000	- 298	- 34	- 5080	-1560
10	18	216	0	0	50,000	"	2.5	126,000	-320	- 10	-5540	-1060

TABLE 10.

CONCENTRATED LOAD.

+ Tension — Compression.

Section	Depth	Area	Shears.		Thrust	H	z	Moments Inch Lbs.	Stresses.			
									Concrete		Steel.	
			Total	Per Sq. In					Upper	Lower	Upper	Lower.
1	26	312	4260	+ 13.6	69600	63500	8.4	534,000	- 516	+ 124	- 8440	+ 1000
2	22	264	4370	+ 16.5	67900	"	3.6	228000	- 382	- 6	- 6600	- 6600
3	20.5	246	4200	+ 17.0	66,600	"	1.2	76000	- 277	- 123	- 5020	- 2980
4	20	240	1255	+ 5.3	65,500	"	1.4	89000	- 116	- 292	- 2660	- 5220
5	19.5	234	145	+ .6	65000	"	1.2	76000	- 123	- 281	- 2940	- 5140
6	19	228	1480	- 6.5	64600	"	.9	57000	- 145	- 267	- 2380	- 2860
7	18.87	226	2130	- 9.4	63100	"	1.0	63500	- 274	- 130	- 5020	- 2060
8	18.37	220	560	- 2.5	63600	"	1.2	76000	- 297	- 117	- 5340	- 2940
9	18.2	218	1120	- 5.2	63500	"	1.2	76000	- 303	- 117	- 5440	- 2960
10	18	216	1010	+ 4.7	63400	"	1.2	76000	- 303	- 117	- 5440	- 2960
11	18.2	218	2950	+ 13.6	63500	"	0	0	- 210	- 210	- 4200	- 4200
12	18.37	220	2240	+ 10.2	63600	"	1.2	76000	- 117	- 297	- 2940	- 5340
13	18.87	226	1230	+ 5.4	64200	"	1.2	76000	- 120	- 290	- 2940	- 5300
14	19	228	1010	+ 4.4	64500	"	1.9	121000	- 68	- 342	- 2332	- 5880
15	19.5	234	560	+ 2.4	65000	"	2.6	165000	- 31	- 203	- 1680	- 6440
16	20	240	2080	- 8.7	65600	"	2.9	184000	- 21	- 387	- 1540	- 6620
17	20.5	246	4370	- 17.7	66500	"	1.6	102000	- 174	- 366	- 4040	- 6760
18	22	264	5940	- 22.4	67,600	"	2.6	165000	- 355	- 55	- 5860	- 1860
19	26	312	6500	- 20.8	69300	"	7.3	464000	- 452	+ 104	- 7560	+ 600

The span is assumed to be restored to its original length during a change of temperature by forces acting horizontally and at a vertical distance, d, above the abutments. The value of this force H, as derived on page 475 of Church's Mechanics, is

$$H = \frac{E I_0 l (t - t_0) \eta}{\Delta s \left[\sum_0^B \left(\frac{y^2}{n}\right) - d \sum_0^B \left(\frac{y}{n}\right) \right]}$$

When η and E have the same meaning as before,

I_0 = moment of inertia at the crown.

l = length of span in inches.

$$d = \frac{\sum\left(\frac{y}{n}\right)}{\sum\left(\frac{1}{n}\right)} = \frac{41.5}{7.118} = 5.82 \text{ feet or } 69.8 \text{ inches.}$$

To find the value of H for a change of one degree $(t-t_0) = 1$. See Table 7.

$$\text{Then } H = \frac{1,400,000 \times 7344 \times 70 \times 12 \times .0000066}{48 \quad (39114-69.8 \times 498) \quad 2} =$$

136 pounds.

Assume a rise and fall of 50° Fahrenheit from the temperature of erection. With a rise of temperature H will rise towards the center and with a fall it acts away from the center. The values of H for the rib shortening in the different conditions of loading are

Dead load, H = 23 x 136 = 3133 pounds.

Uniform live load, H = 23.8 x 136 = 3240 pounds.

Concentrated load, H = 30 x 136 = 4080 pounds, and for changes of temperature H = 50 x 136 = 6800 pounds.

The shears and thrusts are obtained in the same way as before and the bending moment at any section equals the product of H and the vertical distance, Z, from the arch rib to H. The stresses are then computed as before by means of the formulas and are shown in Tables 11, 12, 13 and 14. Combine the stresses algebraically for each section, as in Tables 15, 16 and 17, first adding the stresses due to the loads and those due to rib shortening, and to this add the temperature stresses. The specifications allow 500 pounds compression and 50 pounds tension on the concrete and 10000 pounds stress in the steel, exclusive of temperature stresses. These limits have not been exceeded and hence the bridge is assumed to be safe for any load which it may have to sustain.

In Plate IV is shown a longitudinal section through the center of the bridge, and the centering which was designed to support the loads which would come upon it during construction. Transverse sections are shown at the crown and at the springing line. The bridge was made with enough to allow a twelve foot roadway on which teams could pass, and a three foot walk on each side. The details of the ballustrade are shown on an enlarged scale on the right of the plate.

Plate V is a side elevation of the bridge as it should appear when finished.

TABLE 11.

RIB SHORTENING STRESSES DUE TO DEAD LOAD.

+ Tension — Compression

Section.	Depth.	Area.	Shears.		Thrust.	H	Z	Moments Inch. Lbs.	Stresses.			
									Concrete		Steel.	
			Total	Per Sq. In.					Upper	Lower	Upper	Lower
1	26	312	1430	-4.6	2780	3133	69.8	218400	+124	-138	+1870	-2120
2	22	264	1310	-5.0	2840	"	46.8	146400	+113	-129	+1610	-1920
3	20.5	246	1150	-4.7	2920	"	28.5	89400	+75	-94	+1010	-1380
4	20	240	890	-3.7	3010	"	12.9	40680	+32	-50	+398	-767
5	19.5	234	633	-2.7	3070	"	1.2	3756	+28	-48	+338	-736
6	19	228	532	-23	3100	"	8.0	25200	-37	+17	-567	+169
7	18.87	226	403	-1.8	3110	"	9.0	28200	-44	+24	-629	+230
8	18.37	220	259	-1.2	3120	"	19.8	62400	-83	+63	-1180	+780
9	18.2	218	130	-.6	3125	"	22.4	70320	-97	+77	-1360	+970
10	18	216	0	0	3130	"	23.4	73320	-100	+79	-1410	+980

TABLE 12.

RIB SHORTENING STRESSES DUE TO UNIFORM LIVE LOAD.

Section	Depth	Area	Shears		Thrust	H	Z	Moments Inch Lbs.	Stresses.			
									Concrete		Steel.	
			Total	Per Sq. In.					Upper	Lower	Upper	Lower
1	26	312	1480	-4.7	2870	3240	69.8	226200	+128	-143	+1932	-2220
2	22	264	1360	-5.0	2940	"	46.8	151560	+117	-133	+1660	-1980
3	20.5	246	1190	-4.8	3020	"	28.5	92520	+78	-97	+1050	-1430
4	20	240	920	-3.8	3110	"	12.9	42000	+33	-52	+410	-790
5	19.5	234	655	-2.8	3170	"	1.2	3888	+29	-50	+350	-760
6	19	228	550	-2.4	3200	"	8.0	26040	-38	+17	-590	+175
7	18.87	226	416	-1.8	3210	"	9.0	29160	-45	+25	-650	+240
8	18.37	220	268	-1.2	3230	"	19.8	64200	-86	+65	-1220	+810
9	18.2	218	134	-.6	3235	"	22.4	72600	-100	+80	-1410	+1000
10	18	216	0	0	3240	"	23.4	75720	-104	+82	-1460	+1020

TABLE 13.

RIB SHORTENING STRESSES DUE TO CONCENTRATED LOAD

+ Tension. - Compression.

Section.	Depth.	Area.	Shears.		Thrust.	H	z	Moments. Inch Lbs.	Stresses.			
									Concrete.		Steel.	
			Total	Per Sq. In					Upper	Lower	Upper	Lower.
1	26	312	1870	- 6.0	3620	4080	69.8	285600	+ 162	- 180	+ 2440	- 2800
2	22	264	1710	- 6.5	3700	"	46.8	191400	+ 148	- 168	+ 2100	- 2500
3	20.5	246	1500	- 6.1	3800	"	28.5	116 640	+ 98	- 122	+ 1320	- 1800
4	20	240	1160	- 5.8	3920	"	12.9	52920	+ 42	- 66	+ 520	- 1000
5	19.5	234	825	- 3.5	4000	"	1.2	4800	+ 37	- 63	+ 440	- 960
6	19	228	694	- 3.0	4040	"	8.0	30080	- 48	+ 22	- 740	+ 220
7	18.87	226	525	- 2.3	4050	"	9.0	36720	- 57	+ 31	- 820	+ 300
8	18.37	220	338	- 1.5	4070	"	19.8	27280	- 108	+ 82	- 1540	+ 1020
9	18.2	218	169	- .8	4075	"	22.4	91680	- 126	+ 100	- 1780	+ 1260
10	18	216	0	0	4080	"	23.4	95520	- 131	+ 103	- 1840	+ 1280

TABLE 14.

TEMPERATURE STRESSES FOR RISE OF 50°F

For fall of 50°F the stresses are reversed.

Section	Depth	Area.	Shears.		Thrust.	H	z	Moments. Inch Lbs.	Stresses.			
									Concrete		Steel.	
			Total	Per Sq. In.					Upper	Lower	Upper	Lower.
1	26	312	3120	+ 10.0	6040	6800	69.8	475200	- 295	+ 265	- 4660	+ 4060
2	22	264	2840	+ 10.7	6150	"	46.8	218000	- 280	+ 244	- 4160	+ 3440
3	20.5	246	2500	+ 10.2	6330	"	28.5	194400	- 204	+ 163	- 3000	+ 2200
4	20	240	1940	+ 8.1	6540	"	12.9	88200	- 110	+ 70	- 1670	+ 870
5	19.5	234	1380	+ 5.9	6650	"	1.2	8160	- 105	+ 61	- 1600	+ 735
6	19	228	1160	+ 5.1	6720	"	8.0	54720	+ 36	- 80	+ 366	- 1230
7	18.87	226	870	+ 3.8	6750	"	9.0	61200	+ 51	- 95	+ 500	- 1360
8	18.37	220	560	+ 2.5	6790	"	19.8	134 400	+ 136	- 180	+ 1700	- 2560
9	18.2	218	280	+ 1.3	6810	"	22.4	152 400	+ 166	- 210	+ 2100	- 2960
10	18	216	0	0	6800	"	23.4	159 600	+ 172	- 218	+ 2130	- 3060

TABLE 15.

DEAD LOAD

+ Tension. — Compression.

Section	Shears.				Stresses in Concrete and Steel.															
	Load	Load + Rib Short.	Load + Rib Short + Temp.		Dead Load.				Load + Rib Short.				Load + Rib Short. + Temperature.							
													Rise of Temp.				Fall of Temp.			
					Concrete		Steel.		Concrete		Steel.		Concrete		Steel.		Concrete		Steel.	
			Rise 50°	Fall 50°	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower.
1	+ 5	- .4	+ 9.6	-10.4	- 138	+ 19	-5180	- 340	- 14	- 119	-3310	-2460	- 309	+146	- 7970	+1420	+251	-414	+ 750	-7120
2	+ 7	+ 2.0	+12.7	- 8.7	- 300	- 6	-5200	- 920	- 187	-135	-3590	-2840	-467	+109	- 7750	+ 600	+ 57	-415	- 150	- 7000
3	+ 11	+ 6.3	+16.5	-3.9	-240	- 72	-4320	-1920	-165	-166	-3310	-3300	-369	- 3	-6310	- 1100	- 2	-370	-1110	-6300
4	+ 5.6	+ 1.9	+10.0	-6.2	-185	- 25	-2540	-2860	-153	- 75	- 2142	-3627	-263	- 5	-3812	-2757	- 83	-185	-1272	-5297
5	+ 3.6	+ .9	+ 6.8	- 5.0	-157	-157	-3140	-3140	-129	-205	- 2800	- 3876	- 234	-144	-4402	-3141	- 68	-310	-2067	-5476
6	+ 1.5	- .8	+ 4.3	-5.9	-153	-153	- 3060	- 3060	- 190	-136	-3627	-2891	-154	-216	-3261	-4121	-154	-100	-4857	-2525
7	+ 1.0	- .8	+ 3.0	-4.6	-220	-220	-4400	-4400	-264	-196	- 5029	-4170	-213	-291	-4529	- 5530	-359	-145	-6389	-3670
8	+ 2.0	+ .8	+ 3.3	- 1.7	-222	-220	-4440	-4410	-305	-157	- 5620	- 3660	-169	-337	-3920	-6220	-485	- 21	-8180	-1960
9	+ 1.5	+ .9	+ 2.2	- .4	-229	-229	-4580	-4580	- 326	-152	- 5940	-3610	- 160	-372	-3840	-6570	-536	+ 14	-8900	-1510
10	0	0	0	0	-228	-228	-4560	-4560	-328	-149	-5970	-3580	-156	-367	- 3840	-6640	-546	+23	-9030	-1450

TABLE 16.

UNIFORM LIVE LOAD — 100 Lbs. Per. Sq. Ft.

+ Tension — Compression.

Section.	Shears.				Stresses in Concrete and Steel.															
	Load	Load + Rib Short.	Load + Rib Short + Temp.		Live Load				Load + Rib Short				Load + Rib Short + Temperature.							
					Concrete		Steel		Concrete		Steel.		Rise of Temp.				Fall of Temp.			
													Concrete		Steel.		Concrete		Steel.	
			Rise 50°	Fall 50°	Upper	Lower	Upper	Lower.	Upper	Lower	Upper	Lower.	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower.
1	-2.6	-7.3	+2.7	-17.3	-73	-145	-1780	-4020	+55	-288	+150	-6240	-240	-23	-4510	-2180	+320	-583	+4210	-10800
2	+3.2	-1.8	+8.9	-11.8	-75	-162	-1980	-4500	+42	-295	-320	-6480	-238	-41	-4480	-3040	+286	-575	+3120	-10640
3	+1.8	-3.0	+7.2	-13.2	-43	-205	-2620	-3860	+35	-302	-1570	-5290	-169	-139	-4570	-3090	+198	-506	+630	-8290
4	+9.1	+5.3	+13.4	-2.8	-105	-225	-2460	-4140	-72	-277	-2050	-4930	-182	-207	-3720	-4060	-2	-387	-1180	-5730
5	-4.3	-7.1	-1.2	-13.0	-222	-222	-4440	-4440	-193	-272	-4090	-5200	-298	-211	-5690	-4465	-132	-377	-3255	-6800
6	-3.9	-6.3	-1.2	-11.4	-229	-99	-4180	-2380	-267	-82	-4005	-2205	-231	-162	-3639	-3435	-343	-46	-5235	-3069
7	-.9	-2.7	+1.1	-6.5	-298	-26	-5100	-1380	-343	+1	-5750	-1140	-292	-94	-5250	-2500	-438	+52	-7110	-640
8	-2.0	-3.2	-.7	-5.7	-284	-46	-4900	-1700	-370	+19	-6120	-890	-234	-161	-4420	-3450	-550	+155	-8680	+810
9	+1.9	+1.3	+2.6	0	-298	-34	-5080	-1560	-398	+46	-6490	-560	-232	-164	-4390	-3520	-608	+172	-9450	+1640
10	0	0	0	0	-320	-10	-5540	-1060	-424	+72	-7000	-40	-252	-146	-4870	-3100	-642	+244	-10060	+2090

TABLE 17.

CONCENTRATED LOAD.

+ Tension — Compression.

Section.	Shears.				Stresses in Concrete and Steel.															
	Load	Load + Rib Short	Load + Rib Short + Temp.		Conc. Load.				Load + Rib Short.				Load + Rib Short. + Temperature.							
													Rise of Temp.				Fall of Temp.			
					Concrete		Steel.		Concrete		Steel.		Concrete.		Steel.		Concrete		Steel.	
			Rise 50°	Fall 50°	Upper	Lower	Upper.	Lower.	Upper	Lower	Upper	Lower.	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower.
1	+13.6	+ 7.2	+17.2	− 2.8	−516	+124	−8880	+1000	−354	− 56	−6440	−1800	−649	+209	−11,100	+2260	− 89	−351	−2380	−5860
2	+16.5	+10.0	+20.7	− 0.7	−382	− 6	−6600	−6600	−244	−174	−4500	−9100	−524	+ 70	−8660	−5660	0	−454	−1060	−13260
3	+17.0	+10.9	+21.1	+ 0.7	−277	−123	−5020	−2980	−179	−245	−3700	−4780	−383	− 82	−6700	−2580	− 16	−449	−1500	−7780
4	+ 5.3	− .5	+ 3.1	− 8.6	−116	−292	−2660	−5220	− .74	−358	−2140	−6220	−184	−288	−3810	−5350	− 4	−468	−1270	−7890
5	+ .6	− 2.9	+ 3.0	− 8.8	−123	−281	−2940	−5140	− 86	−344	−2500	−6100	−191	−283	−4100	−5365	− 25	−449	−1765	−7700
6	− 6.5	− 9.5	− 4.4	−14.6	−145	−267	−2380	−2860	−193	−245	−3120	−2640	−157	−325	−4350	−3870	−273	−209	−1890	−2274
7	− 9.4	−11.7	− 7.9	−15.5	−274	−130	−5020	−2060	−331	− 99	−5840	−1760	−280	−194	−5474	−3120	−426	− 48	−7200	−1260
8	−2.5	− 4.0	− 1.5	− 6.5	−297	−117	−5340	−2940	−405	− 35	−6880	−1920	−269	−215	−5180	−4480	−585	+101	−9440	− 220
9	−5.2	− 6.0	− 4.7	− 7.3	−303	−117	−5440	−2960	−429	− 17	−7220	−1700	−263	−227	−5120	−4660	−639	+149	−10180	+ 400
10	+4.7	0	0	0	−303	−117	−5440	−2960	−434	− 14	−7280	−1680	−262	−232	−5150	−4740	−642	+158	−10340	+450
11	+13.6	+14.4	+13.1	+15.7	−210	−210	−4200	−4200	−336	−110	−5980	−2940	−270	−320	−3880	−5900	−546	+56	−8940	− 840
12	+10.2	+11.7	+ 9.2	+14.2	−117	−297	−2940	−5340	−225	−215	−4480	−4320	− 89	−395	−2700	−6880	−405	− 79	−7040	−2620
13	+ 5.4	+ 7.7	+ 3.9	+11.5	−120	−290	−2940	−5300	−177	−259	−3760	−5000	−126	−354	−3160	−6360	−272	−205	−5120	−4500
14	+4.4	+ 7.4	+ 2.3	+12.5	− 68	−342	−2332	−5880	−116	−320	−3072	−5660	− 86	−400	−2416	−6890	−196	−284	−4302	−5294
15	+2.4	+ 5.9	0	+11.8	− 31	−203	−1680	−6440	+ 6	−266	−1240	−6000	− 99	−205	−2840	−5265	+ 67	−371	− 505	−7600
16	−8.7	− 2.9	−11.0	+ 5.2	− 21	−387	−1540	−6620	+ 21	−453	−1020	−7620	− 89	−383	−2690	−6750	+ 91	−563	−150	−9290
17	−17.7	−11.6	−21.8	− 1.4	−174	−366	−4040	−6760	− 76	−488	−2720	−8560	−280	−325	−3720	−6360	+ 87	−692	− 520	−11560
18	−22.4	−15.9	−26.6	− 5.2	−331	− 55	−5860	−1860	−183	−223	−3760	−4360	−463	+ 21	−7920	− 920	+ 61	−503	−320	−8520
19	−20.8	−14.8	−24.8	−4.8	−452	+104	−7560	+ 600	−290	− 76	−5120	−2200	−585	+189	−9780	+1860	−25	−371	− 960	−6860

ESTIMATE OF COST OF BRIDGE.

Concrete in arch ring, 84.34 cu. yds. at $7.50	$587.55
Concrete in spandrel walls, 63.09 cu. yds. at $7.50	473.17
Concrete in walks, 10.5 cu. yds. at $7.50	79.10
Concrete in balustrades, 8.36 cu. yds. at $8.25	68.97
Steel bars, 4" x 1", 22158 lbs. at 4¢	886.32
Steel rods, 1" diameter, 213.6 lbs. at 4¢	8.54
Macadam, 95 ft. at $40000 mile	71.91
Earth fill, 135 cu. yds. at 20¢	27.00
	$2202.56
Add 10% for engineering & incidentals	220.25
TOTAL COST	$2422.81

The price of concrete quoted includes the cost of the forms.

SPECIFICATIONS FOR CONCRETE-STEEL ARCH BRIDGE.

Plans.

The work shall be constructed complete in accordance with the plans, drawings and sections and with these specifications. The specifications and drawings are intended to be co-operative and whatever is called for by either is binding. The work herein described shall be finished in every detail, notwithstanding that every item is not particularly mentioned. The contract price shall be based on these specifications.

Discrepancies.

In case of any discrepancy between the drawings and the figures written on them, the figures shall be taken as correct and in case of discrepancy between the drawings and the specifications, the specifications shall be followed.

Cement.

The cement shall be a standard brand of Portland cement which shall be approved by the engineer. The fineness of the cement shall be such that 99% will pass through a sieve of 50 meshes per lineal inch; at least 90% will pass through a sieve of 100 meshes per lineal inch, and at least

70% will pass through a sieve of 200 meshes per lineal inch.

Samples may be taken from every barrel for testing. Tensile tests shall be made on specimens with an area of one square inch at the breaking section and after being allowed to harden in moist air for 24 hours shall be placed under water and kept there till tested. The sand used in preparing the test specimens shall be the same as that used on the work.

Specimens tested from neat cement shall after 7 days develop a tensile strength of not less than 450 pounds per square inch. Specimens prepared from a mixture of one part cement to three parts sand by weight shall after 7 days develop a tensile strength of not less than 160 pounds per square inch and not less than 220 pounds per square inch after 28 days. Cement made into thin cakes on glass plates shall not crack, scale or warp when allowed to harden in moist air and then immersed in water and maintained there.

All cement shall be kept housed and dry till wanted in the work.

Portland Cement Concrete.

The concrete shall be composed of clean, hard, broken stone, sharp, clean sand and cement mixed in the proportions hereafter specified. If mixing machines are used they shall be of a type approved by the engineer. The ingredients

shall be placed in the machine in a dry state and thoroughly mixed, after which clean water shall be added but not more than the concrete will bear without quaking in ramming. The mixing shall be done as rapidly as possible and be deposited in the work without delay.

If the mixing is done by hand the cement and sand shall first be thoroughly mixed dry in the proportions specified. The stone shall be drenched with water and placed on the mixture. Clean water shall be added and the mass thoroughly mixed till each stone is covered with mortar, and shall be deposited without delay and rammed till all voids are filled.

For the arches and spandrels the concrete shall have the proportions of 1 part Portland cement, 2 parts sand and 5 parts broken stone. The stone shall pass through a 1-1/4 inch ring. The concrete in the ballustrades shall have the proportion of 1 part cement, 2 parts sand and 4 parts broken stone which must pass through a 1/2 inch ring.

Concrete Facing.

The concrete facing shall be composed of 1 part Portland cement and 2-1/2 parts sand and shall have a thickness of 1 inch on all exposed surfaced. The facing and backing must be deposited in the same layer and be well rammed in place at the same time.

Plastering.

No plastering will be allowed on the exposed surfaces of the work.

Sprinkling.

During warm and dry weather all newly built concrete shall be well sprinkled with water for several days or until it is well set.

Mixtures.

The volumes of cement, sand and broken stone in all mixtures of mortar or concrete shall be measured loose.

Connections.

In connecting concrete already set with new concrete the surface shall be cleaned and roughened and mopped with mortar composed of 1 part cement and 1 part sand.

Arches.

Arch concrete shall be laid in transverse sections of the full width of the arch. The length of the sections shall be such that the center section or a pair of intermediate or end sections shall constitute a day's work. Work shall be started at the center section and carried towards the ends, the end sections being laid last.

Drainage.

At the ends of the arch the concrete shall be laid up

so as to drain the water to the center of the arch where a one-inch steel pipe shall pierce the arch. The line of drainage shall be covered with a layer of broken stone.

Steel Ribs.

Steel ribs shall be imbedded in the concrete of the arch. They shall be steel bars 1 inch by 4 inches, spaced equal distances apart and be of the number shown on the plans. The bars shall be about 38 feet in length, thoroughly spliced together. They shall pass through holes of proper size and spaced at the correct distance, in a steel plate 1 inch by 6 inches by 26 inches. Holes 1 inch in diameter shall be bored through the bars through which keys about 4 inches long shall pass. The plate brought to a firm bearing on these keys shall be placed at least 8 inches back in the rock side walls and the hole filled with concrete so as to firmly fasten the bars into the rock. All steel must be free from paint and oil and all scale and rust must be removed before imbedding in the concrete.

Test pieces from the finished bars shall have an ultimate strength of from 60000 to 68000 pounds per square inch and an elastic limit of not less than one half of the ultimate strength. It shall elongate not less than 20% in 8 inches. It must bend cold 180° around a curve whose diameter is equal to the thickness of the piece tested.

Centering.

The contractor shall built the centering according to the plans and take all precautions to make it as unyielding as possible. The lagging shall be dressed to a smooth and uniform surface and this surface shall conform to the lines shown on the drawings. The centering shall not be struck until at least 28 days after the completion of the arch.

Casing.

Substantial and unyielding forms of the proper dimensions shall be built for the spandrel walls and all parts in contact with exposed faces shall have smooth surfaces.

Water-proofing.

After the completion of the arch and spandrel walls, the top of the arch and the inner surface of the spandrels shall be made water-proof by being thoroughly coated with two coats of Portland cement grout.

Fill.

The space between the spandrel walls shall be filled with earth and be thoroughly compacted and finished to the proper grade to receive the macadam and walks.

Sidewalks and Curb.

The space over which the sidewalk is to be laid shall

first be covered with 8 inches of cinders and well compacted. On this shall be laid 5 inches of concrete of the same composition as the spandrels and on this a 1 inch wearing surface composed of 2 parts Portland cement and 3 parts of broken granite or other suitable stone not to exceed 3/8 inch in size. The curb shall be laid with the walk and in the same manner.

Pavement.

The pavement shall be macadam made of 8 inches of broken stone rolled down to 6 inches, as shown on the plans.

Balustrades.

The balustrades shall be of the form and dimensions shown on the plans and shall be brought to a true alignment.

Erection.

The contractor shall employ suitable labor for each kind of work. He shall furnish all appliances required on the work and shall assume all risks for loss or damage during construction.

Inspection.

All material furnished by the contractor shall be subject to the inspection and approval of the engineer.

Removal of Old Bridge.

The contractor shall remove the bridge at present

occupying the site.

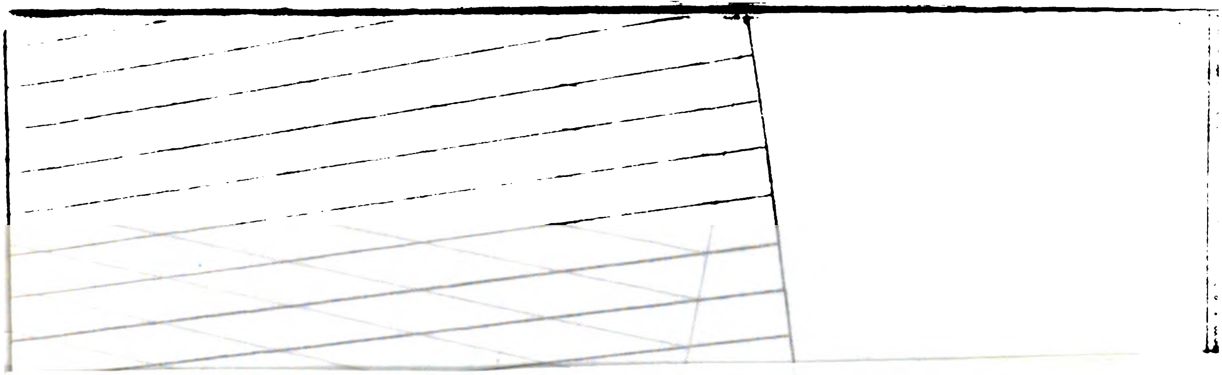
Cleaning up.

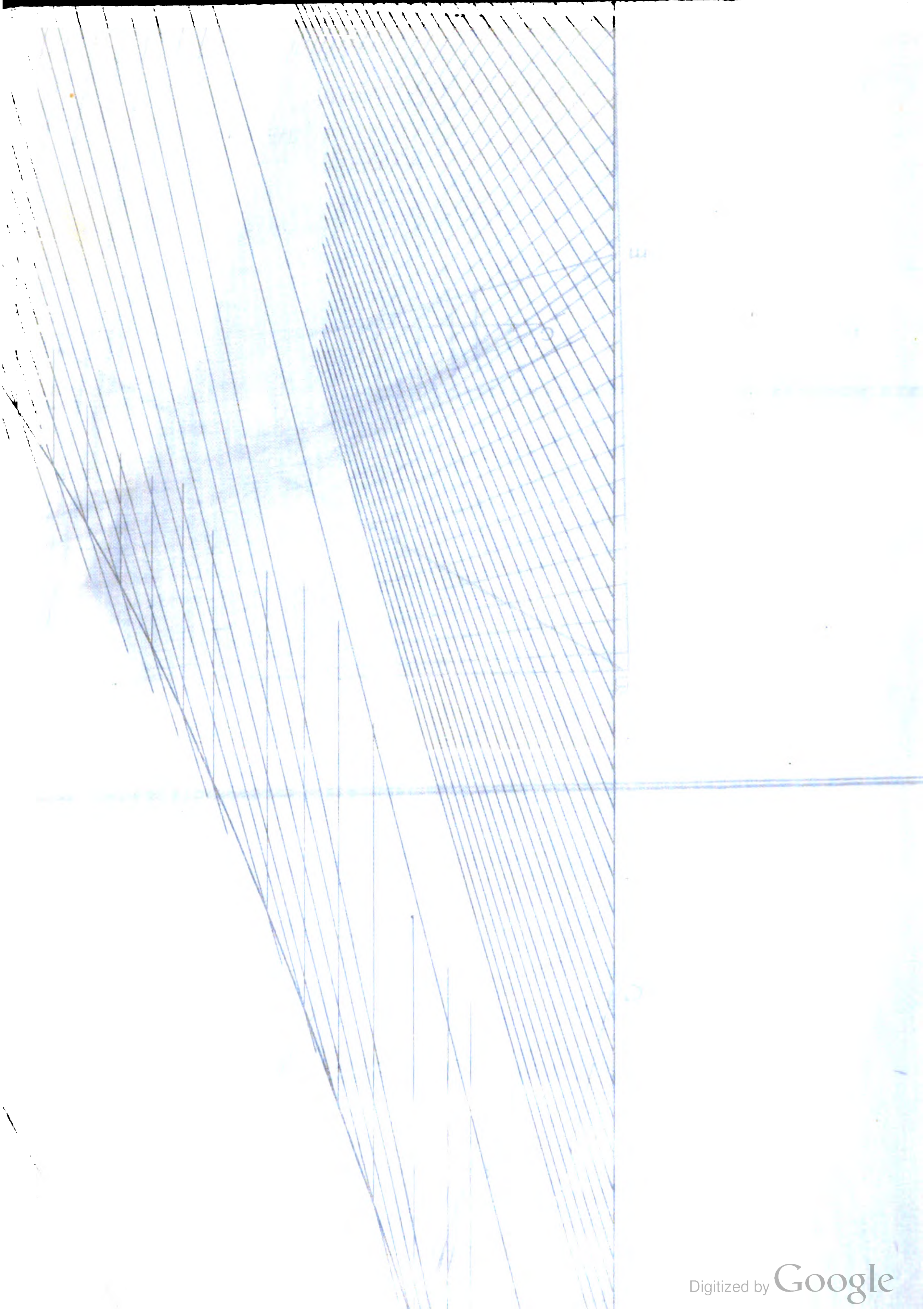
On completion of the bridge and before acceptance, the contractor shall remove all temporary work and rubbish from the site.

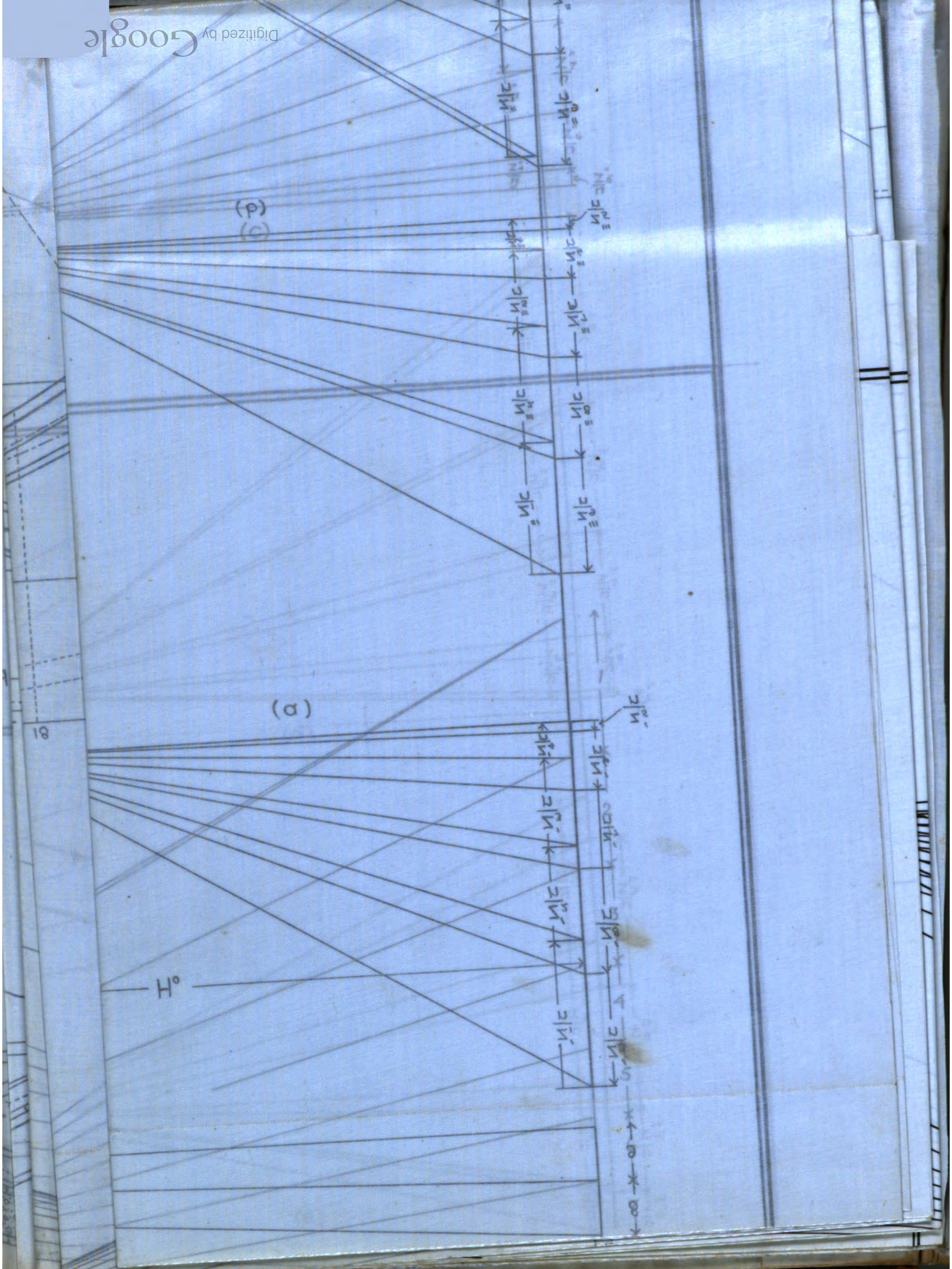
APPROVED:

F. E. Turneaure

Prof. of Bridge & Sanitary Eng'g.



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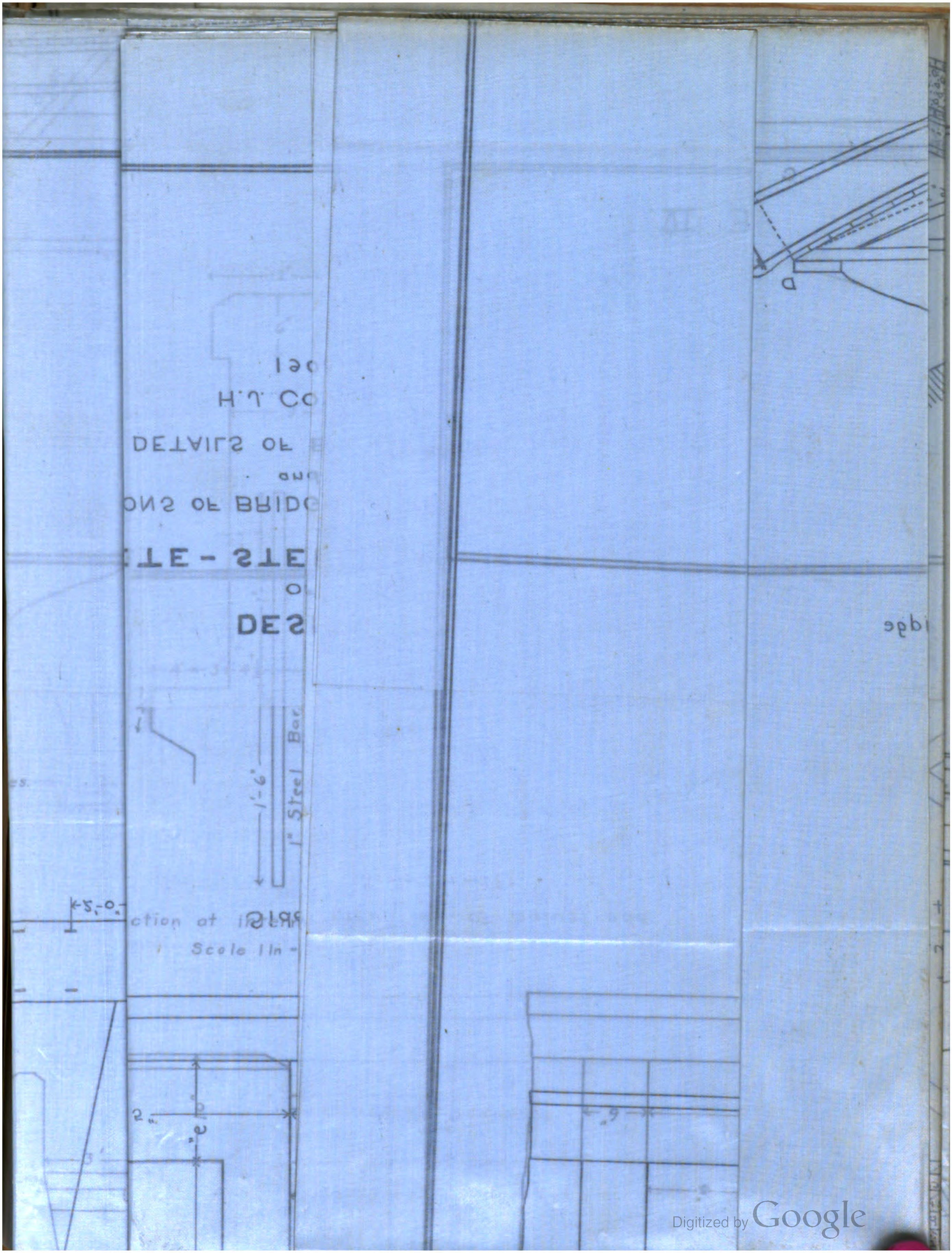
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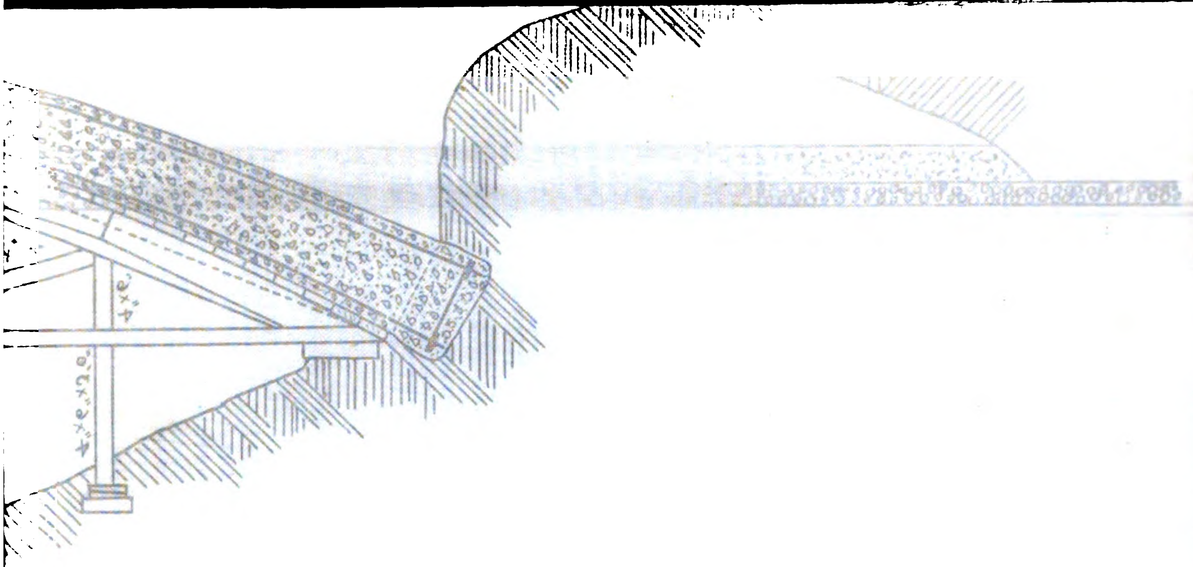
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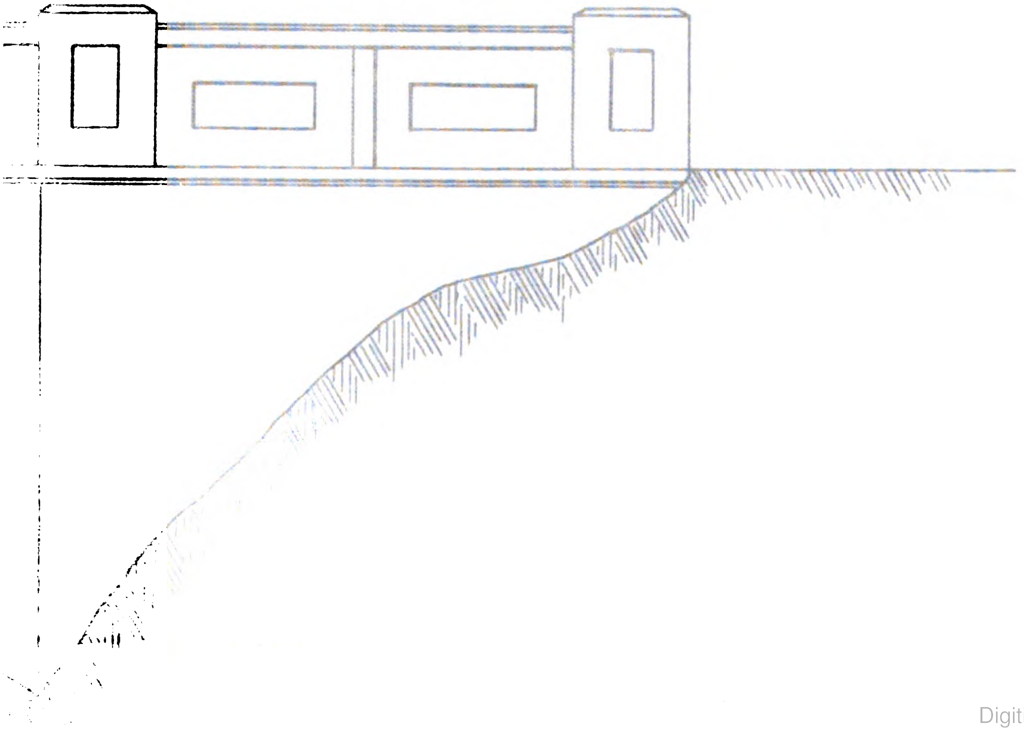

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